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CP violation in unpolarized $e^+e^- \to$ charginos at one loop level

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We study CP violation in $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-$ in the framework of the MSSM. Though the cross section of this process is CP-even at the tree level even for polarized electron-positron beams, we show that it contains a CP-odd part at the one loop order and there are CP-odd observables that can in principle be measured even using *unpolarized* electron-positron beams. The relevant diagram calculations are briefly discussed and the results of selected (box) diagram computations are shown.

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I. INTRODUCTION

The complex phases of the higgsino and gaugino mass parameters in the Minimal Supersymmetric Standard Model (MSSM [1, 2]) allow for CP violation at low orders of perturbation theory, without invoking the Cabibbo–Kobayashi–Maskawa matrix or the Higgs sector. If the phases are significant, one may expect experimental evidence of CP violation that does not fit the explanation within the (non-supersymmetric) Standard Model, leave alone the consequences for CP-conserving processes. It has long been known that these phases, if $\mathcal{O}(1)$, could lead to values for the electron and neutron electric dipole moments that would violate the experimental bounds unless the superparticles had masses of $\mathcal{O}(\text{TeV})$ or higher [3]. However, it has recently been realized that there could well be cancellations among various contributions to such CP-violating effects [4, 5], such that the experimental constraints are respected, even with some phases of $\mathcal{O}(1)$ and some superparticles light.

The couplings with potentially CP-violating phases affect many cross sections and rates. However, the most informative way to study such couplings would be in some CP-odd observable that would be accessible in future experiments. In the light of the International Linear Collider project [6], it is natural to consider the products of e^+e^- annihilation. The chargino pair creation

$$e^+ + e^- \to \tilde{\chi}_i^+ + \tilde{\chi}_j^- \tag{1.1}$$

then immediately comes to mind. At tree level the neutralino couplings do not enter in the amplitude and the only CP-violating phase that enters is ϕ_μ due to the higgsino mass parameter

$$\mu \equiv |\mu| e^{i\phi_\mu}. \tag{1.2}$$

This phase is indeed accessible at tree level if one in mixed events ($i \neq j$) measures the transverse polarization of one of the charginos [7, 8, 9]. There is also an additional CP violating effect in chargino decays, at the one-loop level [10]. However, if one does not consider the decay of a final-state chargino, the tree-level cross section of the above process conserves CP (in the $m_e = 0$ limit) [7], even if one considers polarized electron-positron beams [11].

At the same time, there is no physical *symmetry* which whould prohibit the cross section from acquiring a CP-odd part: the result of [11] is mainly dictated by the V-A structure of the tree-level couplings (see the general discussion of the effective form factors given in [12, 13]). Since by the very construction MSSM is renormalizable and the tree-level cross section is CP even, any non-vanishing CP-odd contribution should at one loop be *finite* — that is the logic of renormalization and that is why many regularization problems [14] drop out for this effect (see Sec. III B).

Typically, to build a (scalar) CP-odd observable in a $2 \to 2$ process one has to employ spin (polarization) of one of the particles in addition to the particle momenta, since any scalar product of momenta is even under C and P [15, 16]. However, the careful analysis in Sec. II shows, that when the final chargino mass indices are different,[1] their interchange should also be accounted for and a CP-odd observable is easily constructed out of unpolarized cross-sections. So, the CP-violation may in principle be observed in the reaction (1.1) without any spin detection and with unspecified polarization for the initial beams. This is the main result of the present paper.

While the identified effect is radiatively induced, and thus of $\mathcal{O}(\alpha)$, there could be enhancements due to factors $\tan\beta$ or $\cot\beta$. In any case, we think an independent CP-violating effect is worth attention, if some kind of supersymmetry should be realized in nature. In particular, it may provide information on whether the chargino sector contains more than two mass states, and information on the neutralino sector, including the phase of the U(1) gaugino mass parameter, M_1, via the $W^\pm\tilde{\chi}_i^\mp\tilde{\chi}_k^0$ couplings.

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[1] We use a “mass index”, taking values 1 and 2, to distinguish the two chargino mass states.

Following many authors we work within the simplest version of unconstrained MSSM making no assumptions about the symmetry breaking mechanisms [17], neither do we impose any constraints on the CP-violating phases. The R-parity and the lepton flavour violation is not permitted, though, as noted in [11], the modification for less constrained models can easily be done. Besides, just to simplify sample calculations we assume that all slepton masses are large,[2] and, of course, neglect everything proportional to the electron mass. We do not calculate the one-loop cross section here, neither do we give a review of the magnitude of the CP-odd observables in various parameter points. Instead, we pick a specific parameter set that allows us to neglect certain diagrams and show that the effect is indeed non-zero at the one-loop order.

II. CP TRANSFORMATION OF THE CROSS SECTION AND THE CP-ODD OBSERVABLE

Let us consider chargino production in e^+e^- annihilation allowing for polarized initial beams:

$$e^+(p_1, P_+) + e^-(p_2, P_-) \to \tilde{\chi}_i^+(k_1) + \tilde{\chi}_j^-(k_2), \qquad (2.1)$$

where $P^\mu_\pm$ are the positron and electron polarization four-vectors [19] (see also [20]). The crucial point here is that for $i \neq j$ the charginos do not form a particle-antiparticle pair. Hence, while the initial state (for suitably chosen polarizations, $P_+ \leftrightarrow P_-$) is in the c.m. frame odd under charge conjugation, the final state has no such symmetry. We shall take a closer look at this.

The C and P unitary operators act in Fock space and transform the creation operators as [21, 22]: $\mathrm{P}a^\dagger(\boldsymbol{p}, \sigma, n)\mathrm{P}^{-1} = \eta_n a^\dagger(-\boldsymbol{p}, \sigma, n)$; $\mathrm{C}a^\dagger(\boldsymbol{p}, \sigma, n)\mathrm{C}^{-1} = \xi_n a^\dagger(\boldsymbol{p}, \sigma, n^c)$, where η and ξ are the intrinsic space inversion and charge-conjugation phases (parities), $\boldsymbol{p}$ is the three-momentum, σ labels spin components, while n and n^c refer to other quantum numbers[3] for particle and antiparticle, respectively. Hence, under P, C, and CP conjugation the S-matrix element $\langle \tilde{\chi}_i^+(\boldsymbol{k}_1), \tilde{\chi}_j^-(\boldsymbol{k}_2)|S|e^+(\boldsymbol{p}_1, P_+), e^-(\boldsymbol{p}_2, P_-)\rangle$ of the process (2.1) gets transformed into (up to a phase that does not affect the cross section):

$$\text{P:} \quad \langle \tilde{\chi}_i^+(-\boldsymbol{k}_1), \tilde{\chi}_j^-(-\boldsymbol{k}_2)|S|e^+(-\boldsymbol{p}_1, P_+), e^-(-\boldsymbol{p}_2, P_-)\rangle;$$

$$\text{C:} \quad \langle \tilde{\chi}_i^-(\boldsymbol{k}_1), \tilde{\chi}_j^+(\boldsymbol{k}_2)|S|e^-(\boldsymbol{p}_1, P_+), e^+(\boldsymbol{p}_2, P_-)\rangle;$$

$$\text{CP:} \quad \langle \tilde{\chi}_j^+(-\boldsymbol{k}_2), \tilde{\chi}_i^-(-\boldsymbol{k}_1)|S|e^+(-\boldsymbol{p}_2, P_-), e^-(-\boldsymbol{p}_1, P_+)\rangle. \qquad (2.2)$$

Thus, the cross section for the P-conjugated process can be obtained by the change of sign of the particle three-momenta: $\boldsymbol{p}_{1,2} \leftrightarrow -\boldsymbol{p}_{1,2}$, $\boldsymbol{k}_{1,2} \leftrightarrow -\boldsymbol{k}_{1,2}$; the C-conjugation amounts to the following substitution in the cross section: $\boldsymbol{p}_1 \leftrightarrow \boldsymbol{p}_2$, $\boldsymbol{k}_1 \leftrightarrow \boldsymbol{k}_2$, $m_i \leftrightarrow m_j$, $P_+ \leftrightarrow P_-$; and the CP-transformation results in the change: $\boldsymbol{p}_1 \leftrightarrow -\boldsymbol{p}_2$, $\boldsymbol{k}_1 \leftrightarrow -\boldsymbol{k}_2$, $m_i \leftrightarrow m_j$, $P_+ \leftrightarrow P_-$.

To find candidates for CP-sensitive observables, let us write the cross section as

$$d\sigma = d\sigma_0 + (\text{terms linear in } |\boldsymbol{P}_\pm|) + (\ldots)|\boldsymbol{P}_-||\boldsymbol{P}_+|,$$

where $d\sigma_0$ does not depend on polarization vectors and will be referred to as the *unpolarized* part. Due to Poincaré invariance $d\sigma_0$ may depend only on masses m_i, m_j and on two independent scalar variables, say, on Mandelstam's $s \equiv (p_1 + p_2)^2$ and $t \equiv (p_1 - k_1)^2$. The latter do not change under C or P, so the CP-transformation for the unpolarized cross-section is reduced to the interchange of the masses in the resulting formula[4]. Therefore, for equal-mass fermions in the final state ($i = j$) the unpolarized cross section is always P-, C- and CP-even[5]. *In contrast, if the chargino species are different, CP-violating terms can arise even in the unpolarized cross-section.* That is the effect we will consider here, so unless otherwise stated the final-state chargino masses are taken non-equal.

Calculations show that the tree-level cross section (polarized and unpolarized) of the process (2.1) is CP even [11], but, as we shall see, CP-odd terms do arise in the one-loop contributions. Therefore, a natural experimental observable to consider is the ratio

$$\frac{d\sigma_0^{\rm odd}}{d\sigma_0}\,, \qquad (2.3)$$

where $d\sigma^{\rm odd}$ is the CP-odd part of the corresponding cross-section:

$$d\sigma_0^{\rm odd} = \frac{1}{2}\Big[d\sigma_0 - d\sigma_0^{\rm CP}\Big], \quad d\sigma_0^{\rm CP} \equiv d\sigma_0\Big|_{m_i \leftrightarrow m_j}. \qquad (2.4)$$

As just mentioned, the CP violation first enters at one loop, thus, to estimate the effect one should caculate $d\sigma_0^{\rm odd}$ at the one-loop level. On the other hand, in most of the kinematical regions far from any resonance, one can expect (see, e.g, [23, 24, 25, 26]) that the tree-level

[2] One could refer to the parameter space area around the so-called SPS 2 benchmark point [18], but remember that the latter classification assumes an mSUGRA breaking mechanism with no CP-violating phases.

[3] Like charge, chargino mass index, etc. Following the most common convention, we treat the *positive* chargino as particle, its antiparticle is, of course, the negative chargino with the *same* mass (mass index).

[4] Of course, the coupling constants at vertices with charginos should be considered as functions of the chargino masses, or, better, the mass indices i, j.

[5] The famous forward-backward asymmetry term in the *unpolarized* cross-section of, say, $e^+e^- \to \mu^+\mu^-$ scattering, which is often referred to as parity violating, in fact only indicates the presence of a parity violating term in the interaction, the unpolarized cross-section itself being, of course, P-even.

gives a reasonable approximation to $d\sigma_0$ in the denominator of Eq. (2.3). So, we will deal only with the ratio

$$\frac{d\sigma_0^{\rm odd}\big|_{\rm 1\ loop}}{d\sigma_0\big|_{\rm tree}}\ . \tag{2.5}$$

In the following Sections we discuss the diagrams that (may) contribute to this observable and provide some sample calculations.

III. DIAGRAMS

The MSSM spectrum and Lagrangian are reviewed by many authors (e.g. [1, 2]), we use the Feynman rule collections of [27, 28]. Following the latter article, we work in 't Hooft–Feynman gauge [29, 30], though for more involved loop calculations other gauge choices may be preferable [14, 31]. When drawing diagrams, we found it convenient to indicate sparticles by double lines. Due to R-parity conservation,[6] the total number of such lines attached to each vertex should be even. Following [33], we do not indicate the (double) fermion line direction for the neutralino and choose a convenient fermion flow for each diagram.

A. Tree diagrams

We need the tree-level cross section to normalize the observable (2.5). The graphs contributing to the tree amplitude $\mathcal{M}_{\rm tree}$ are drawn[7] in Fig. 1. They are: s-channel Higgs (and the unphysical Goldstone), photon and Z exchanges, and t-channel sneutrino exchange.

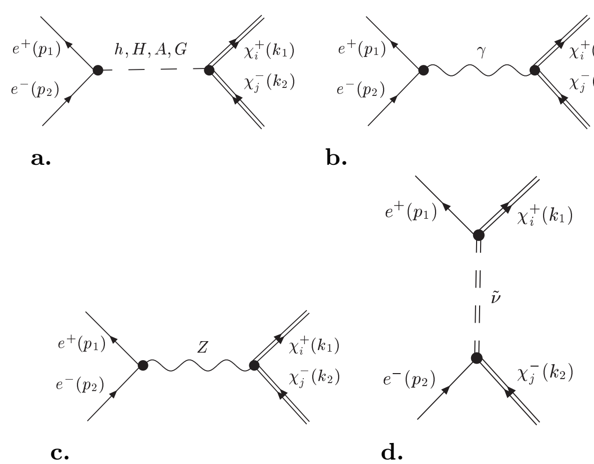

FIG. 1: Tree-level diagrams: superpartners of ordinary particles are pictured by double lines.

[6] Recently the R-parity non-conserving extensions of the MSSM started to attract attention (see e.g. [32]), however here we do not consider these cases.

[7] Diagrams are drawn by *JaxoDraw* tools [34].

The Higgs (Goldstone) exchanges can be dropped since their couplings are proportional to m_e, while γ exchange is absent since the final-state charginos have different masses and there is no non-diagonal coupling with the photon in the MSSM (this is a requirement of gauge invariance and renormalizability). Finally, to make sample loop calculations simpler, we assume that all sleptons are heavy and, hence, only the Z-exchange contributes at tree level.

The differential cross section (in the c.m. system) is

$$\frac{d\sigma}{d\Omega}=\frac{\beta}{64\pi^2 s}|\mathcal{M}|^2\ ,\quad \beta\equiv\frac{|\boldsymbol{p}_{\rm out}|}{|\boldsymbol{p}_{\rm in}|}\ , \tag{3.1}$$

and the direct calculation for unpolarized Z-exchange amplitude gives (cf. [11]):

$$\begin{aligned}&|\mathcal{M}_{Z,\ \rm tree}|^2\\ &\ =\ \chi^2\Big((g_V^2+g_A^2)\big\{|G_V|^2[\mathcal{A}-2(m_i-m_j)^2/s]\\ &\qquad +|G_A|^2[\mathcal{A}-2(m_i+m_j)^2/s]\big\}\\ &\qquad -4g_Vg_A(G_V^*G_A+G_VG_A^*)\beta\cos\theta\Big),\end{aligned} \tag{3.2}$$

where $s=(p_1+p_2)^2$, m_i, m_j are the chargino massess, θ is the scattering angle, and

$$\chi=\left(\frac{g}{4\cos\theta_W}\right)^2\frac{s}{s-M_Z^2},\quad \mathcal{A}=2-\beta^2\sin^2\theta.$$

The Zee (reduced) couplings are: $g_V=1-4\sin^2\theta_W$, $g_A=-1$, and we use $G_V\equiv G_{V\,j,i}$ and $G_A\equiv G_{A\,j,i}$ to abbreviate the $Z\chi\chi$ coupling constants:

$$\begin{aligned}L_{Z\chi\chi}\ &=\ \frac{g}{4\cos\theta_W}\bar\Psi_{\chi_j}\gamma^\rho\Big\{\Big[2\delta_{kj}\cos2\theta_W\\ &\qquad +U_{k1}U^\dagger_{1j}+V_{j1}V^\dagger_{1k}\Big]\\ &\qquad +\gamma^5\Big[U_{k1}U^\dagger_{1j}-V_{j1}V^\dagger_{1k}\Big]\Big\}\Psi_{\chi_k}Z_\rho\\ &\equiv\ \frac{g}{4\cos\theta_W}\bar\Psi_{\chi_j}\gamma^\rho\Big\{G_{V\,k,j}\\ &\qquad +\gamma^5G_{A\,k,j}\Big\}\Psi_{\chi_k}Z_\rho\end{aligned} \tag{3.3}$$

(note, that the first mass index of $G_{V\,k,j}$ and $G_{A\,k,j}$ refers to the mass of the *annihilated particle*, which is the *positive* chargino). The matrices U and V diagonalize the chargino mass matrix M_χ:

$$\begin{aligned}M_\chi&=\begin{pmatrix}M_2 & \sqrt2 m_W\sin\beta\\ \sqrt2 m_W\cos\beta & \mu\end{pmatrix}\\ U^*M_\chi V^\dagger&=\begin{pmatrix}m_{\chi_1} & 0\\ 0 & m_{\chi_2}\end{pmatrix},\quad 0<m_{\chi_1}<m_{\chi_2}.\end{aligned} \tag{3.4}$$

The SU(2) gaugino mass parameter M_2 can always be chosen real, while μ (as well as the U(1) gaugino mass parameter M_1 appearing in the neutralino mass matrix) is in general complex quantity.

According to Sec. II, under the CP-transformation $G_{V(A)\,j,i} \leftrightarrow G_{V(A)\,i,j}$, $m_i \leftrightarrow m_j$. On the other hand, the hermiticity of the Lagrangian enforces the relation

$$G_{V(A)\,j,i} = G^*_{V(A)\,i,j}, \tag{3.5}$$

so Eq. (3.2) is clearly CP-even.

If sneutrino exchange is not neglected, the cross section consist of the squared graphs terms (Fig. 1 **c**, **d**) and an interference term. Each of them turns out to be CP-even [11].

B. Loops

The complete list of one-loop (prototype) graphs contributing to the cross section can be found in [24]. The fact that the tree-level cross section is CP even makes it evident that many of the $d\sigma_0$ one-loop corrections cancel in $d\sigma_0^{\rm odd}$ (2.4), the numerator of (2.5). Indeed, the external wave function renormalization is multiplicative, the propagator corrections result just in a propagator mass shift,[8] therefore we do not need to calculate the two-point functions and, hence, neither Faddeev–Popov ghosts nor coloured particles will be involved. In other words, there are only two types of one-loop corrections that may contribute to $d\sigma_0^{\rm odd}$: box diagrams and the tree diagrams from Fig. 1 with a triangle loop instead of one of the vertices. Before we take a closer look at the box diagrams (we do not compute the triangle vertex corrections here), it is necessary to say a couple of words about the ultraviolet and infrared behaviour of $d\sigma_0^{\rm odd}$ at the one-loop order.

As mentioned in the Introduction, $d\sigma_0^{\rm odd}$ must be UV finite, since it vanishes at tree level: otherwise the counterterms required would mirror the tree level CP-odd contribution. So, no infinite (UV-divergent) counterterms are required. In fact, one can also see that any *finite* counterterm just results in corrections to the tree level vertices in Fig. 1. In particular, in Eq. (3.2) only $g_{V,A}$ and $G_{V,A}$ may get modified, and, since Eq. (3.5) should always hold, the result will still be CP-even and no contributions to $d\sigma_0^{\rm odd}$ will arise. One can easily check that the unpolarized cross section with sneutrino exchange will also be unaffected by counterterms. This relates also to finite counterterms which may be required to restore the symmetries violated by regularization[9] in the so-called algebraic renormalization approach [14]. So, at least for the unpolarized cross section, we should not worry about the renormalization scheme (we assume that the on-shell normalization conditions are used) and the standard dimensional regularization will be adequate at the one loop order: all divergent pieces must cancel in $d\sigma_0^{\rm odd}$.

The situation with infrared (IR) finiteness is slightly more complicated: there are many loops with massless particles inside. However, according to [14], all the IR singularities that appear at any loop order in our amplitude are of the standard type, namely, they arise due to the soft photons and cancel when real bremsstrahlung is accounted for. On the other hand, the bremsstrahlung photon emission from the tree diagram results in just an overall factor for the corresponding amplitude. Since the tree amplitude is CP-even, we conclude that $d\sigma_0^{\rm odd}$ is free of IR singularities.

Each possible box diagram turns out to be UV-finite just by power counting. Since we assume heavy sleptons, any box with a slepton line can be neglected. The only box diagrams that may contribute to Eq. (2.5) in this limit are drawn in Fig. 2. Those are the only graphs whose contribution to $d\sigma_0^{\rm odd}$ we shall evaluate numerically. Analytical results for the coefficients of the box type Passarino–Veltman-like functions presented in the next section ensure that the CP-odd contribution from box diagrams can not be completely cancelled by graphs with triangle loop corrections and, hence, the CP-violation is indeed present in the unpolarized cross section.

IV. NUMERICAL ESTIMATES

Loop amplitudes are conveniently evaluated in terms of Passarino–Veltman functions [35]. In [24] the cross section of the process (1.1) was parametrized in terms of those functions and calculated in various parameter points. However, the latter results were obtained assuming a CP-invariant theory (real couplings) and (to make the results compact) the reduction to scalar Passarino–Veltman functions was not done. Since only the scalar functions can be considered independent (differ from each other by singularity pattern) we performed this reduction in our formulae.

At one loop order the cross section is defined by

[8] It is a bit more tricky if one sticks to precise one-loop order and does not allow for the Dyson resummation in the propagators. Then each of the tree graphs (Fig. 1) acquires different functional (CP-even) multiplier and the structure of the tree-level result [11] ensures that CP-odd terms cannot arise. We do not demonstrate it here as we discard the sneutrino exchange graph, and, therefore, will get a multiplicative correction anyway.

[9] There are, however, no such simple arguments for *polarized* amplitudes, as one of the potential CP-odd terms in this case is cancelled due to the tree level SUSY relation between chargino and sneutrino couplings [11]. The symmetry-restoring counterterms may, in general, violate this relation and therefore can give an additional CP-odd term. We shall not discuss it here.

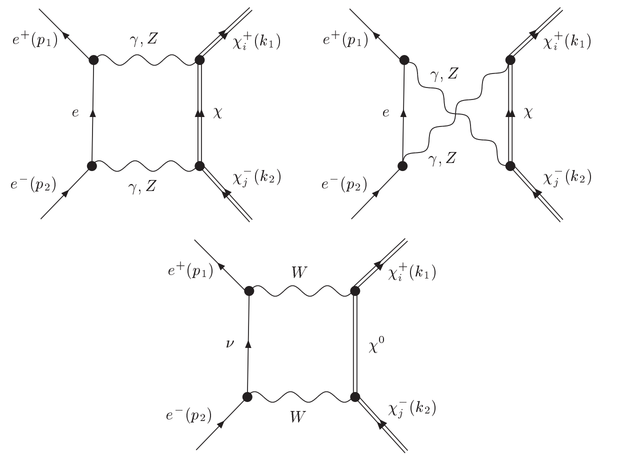

FIG. 2: Box diagrams without slepton lines: all chargino and neutralino mass eigenstates contribute.

Eq. (3.1) with

$$\left|\mathcal{M}^2\right|_{1\,\text{loop}} = \mathcal{M}^*_{\text{tree}}\mathcal{M}_{1\,\text{loop}} + \mathcal{M}_{\text{tree}}\mathcal{M}^*_{1\,\text{loop}} \ ,$$

and, since we assume heavy sleptons, the tree amplitude $\mathcal{M}_{\text{tree}}$ contains only the s-channel Z-exchange graph of Fig. 1. Direct calculations [36] show that the CP-odd part of $\left|\mathcal{M}^2\right|_{1\,\text{loop}}$ accquire four-point ("box") integral contributions. In particular, for the Z-exchange (uncrossed and crossed) box diagrams of Fig. 2, after reduction to scalar integrals one obtains (the subscript "D" refers to terms proportional to genuine box diagram functions, as defined below):

$$\begin{aligned}
&\left|\mathcal{M}^2\right|_{\substack{\text{CP-odd,}\\ \text{Z-box, D}}} \\
&= \frac{1}{(2\pi)^4} 2\text{Re}\Big[\frac{ig^6 m_i m_j}{(4\cos\theta_W)^6}(G_{A\,ij}G_{V\,ji} - G_{A\,ji}G_{V\,ij}) \\
&\quad \times\big\{g_A(g_A^2+3g_V^2)m_Z^2(G_{V\,ii}I_{i;ji} + G_{V\,jj}I_{j;ji}) \\
&\quad + g_V(3g_A^2+g_V^2)\big[(2m_i^2 - m_Z^2 - 2t)G_{A\,ii}I_{i;ji} \\
&\qquad + (2m_j^2 - m_Z^2 - 2t)G_{A\,jj}I_{j;ji}\big] \\
&\quad + g_A(g_A^2+3g_V^2)m_Z^2(G_{V\,ii}I^{\text{cr}}_{i;ji} + G_{V\,jj}I^{\text{cr}}_{j;ji}) \\
&\quad - g_V(3g_A^2+g_V^2)\big[(2m_i^2 - m_Z^2 - 2u)G_{A\,ii}I^{\text{cr}}_{i;ji} \\
&\qquad + (2m_j^2 - m_Z^2 - 2u)G_{A\,jj}I^{\text{cr}}_{j;ji}\big]\big\}\Big]. \qquad (4.1)
\end{aligned}$$

Here, as above, m_i, m_j are the chargino masses ($i, j = 1, 2$), and couplings are defined in Sec. III A. From the tree diagram there is a coupling g_V or g_A at the Zee vertex, and a $G_{V\,ji}$ or $G_{A\,ji}$ at the $Z\chi_i\chi_j$ vertex, whereas the box diagrams contribute two Zee couplings (g_V^2, g_A^2 or $g_V g_A$), and two $Z\chi\chi$ vertices, one of which will be diagonal in mass index ($G_{V\,ii}$, $G_{A\,ii}$, $G_{V\,jj}$ or $G_{A\,jj}$), and one will be non-diagonal ($G_{V\,ij}$, $G_{A\,ij}$, $G_{V\,ji}$ or $G_{A\,ji}$). The two non-diagonal $Z\chi\chi$ couplings factor out as the combination

$$G_{A\,ij}G_{V\,ji} - G_{A\,ji}G_{V\,ij} = 2i\,\text{Im}\,G_{A\,ij}G_{V\,ji}. \qquad (4.2)$$

This quantity is shown in Fig. 3, for the set of parameters:

$$|\mu| = 300 \text{ GeV}, \quad M_2 = 200 \text{ GeV}. \qquad (4.3)$$

We note that the quantity (4.2) increases with decreasing values of $\tan\beta$.

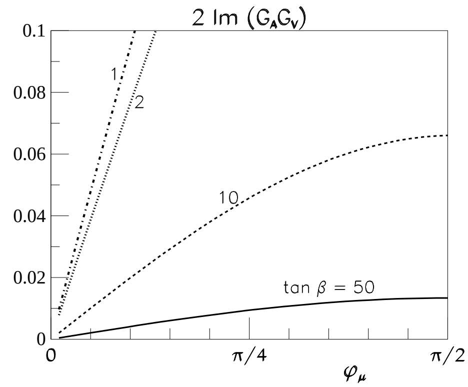

FIG. 3: The couplings of Eq. (4.2) vs. ϕ_μ for various values of $\tan\beta$.

The integrals I and I^{cr} of Eq. (4.1) are the Passarino–Veltman scalar four-point functions which correspond to "normal" and "crossed" box diagrams in Fig. 2, respectively:

$$\begin{aligned}
I_{k;ij} &\equiv D(p_1, p_2, -k_2, -k_1, m_Z, 0, m_Z, m_{\chi_k}) \\
I^{\text{cr}}_{k;ij} &\equiv D(p_1, p_2, -k_1, -k_2, m_Z, 0, m_Z, m_{\chi_k}) \ ,
\end{aligned}$$

where, following [35],

$$\begin{aligned}
&D(l_1, l_2, l_3, l_4, m_1, m_2, m_3, m_4) \\
&\equiv \int \mathrm{d}^4 q\big\{(q^2 - m_1^2)[(q+l_1)^2 - m_2^2] \\
&\qquad \times[(q+l_1+l_2)^2 - m_3^2] \\
&\qquad \times[(q+l_1+l_2+l_3)^2 - m_4^2]\big\}^{-1} \qquad (4.4)
\end{aligned}$$

(in numerical calulations one may favour a more symmetric loop momentum assignment which permits consistency cross tests [36]). Analogous, though more cumbersome, pieces follow from the box diagram with W-exchange (the D-pieces of the γZ-exchange box diagrams cancel) and one can check that all these four-point integral contributions do not cancel each other. Besides, two- and three-point integrals (denoted B and C in [35]) also appear after reduction of tensor box integrals stemming from the diagrams in Fig. 2. What is essential, is that while the graphs with triangle vertex corrections may contribute B and C (and, possibly, A — the one-point) functions to $d\sigma_0^{\text{odd}}$, the D function can never appear in triangle diagrams. As the function D cannot be constructed out of A, B, C integrals and rational functions, we may conclude that $d\sigma_0^{\text{odd}}$ is non-zero at the one-loop order.

Even assuming heavy sleptons the total box diagram contribution to $d\sigma_0^{\rm odd}$ is too awkward[10] to provide here the complete formulae. Instead, to give an idea about the orders of magnitude, we shall provide some plots. We stress once more that the triangle loop corrections to the tree-level vertices are not accounted for, therefore the numbers given are purely illustrative. Below, the ratio (2.5) (with the amplitude $\mathcal{M}_{1\ \rm loop}$ built solely of the diagrams in Fig. 2) is plotted as a function of the CP-violating phase ϕ_μ of Eq. (1.2), and for simplicity the U(1) gaugino mass parameter appearing in the neutralino mass matrix is taken to be real: $M_1 = 250$ GeV . The absolute values of the remaining chargino and neutralino mass matrix parameters are given by Eq. (4.3).

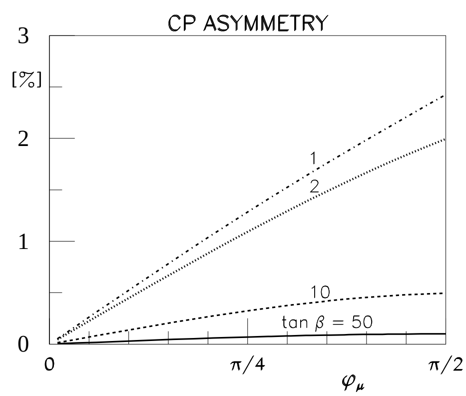

FIG. 4: The ratio (2.5) at various values of ϕ_μ and $\tan\beta$. The polar scattering angle is $\theta = \pi/3$ while $\sqrt{s} = 600$ GeV.

We start this little numerical digression by showing in Fig. 4 the asymmetry resulting from the box diagrams (for the numerical work, we used the `LoopTools` [37, 38] package), as a function of ϕ_μ, for $\sqrt{s} = 600$ GeV, $\cos\theta = 0.5$ and a few values of $\tan\beta$. As anticipated, for $|\phi_\mu| \ll 1$, the effect is linear in ϕ_μ. Also, we note that the shape of these curves (i.e., dependence on ϕ_μ and $\tan\beta$) is essentially given by the coupling constants (4.2) and shown in Fig. 3.

When the energy increases, the effect is reduced, as illustrated in Fig. 5, where we show similar plots for $\sqrt{s} = 800$ GeV. The CP violation is related to the fact that the two charginos will have different velocities (due to different masses). At high energies, the difference in masses plays a lesser role.

The asymmetry demonstrates a smooth behaviour with respect to the polar angle (see Fig. 6).

Since the effect somehow is due to the fact that the two chargino mass states are different, one might think that

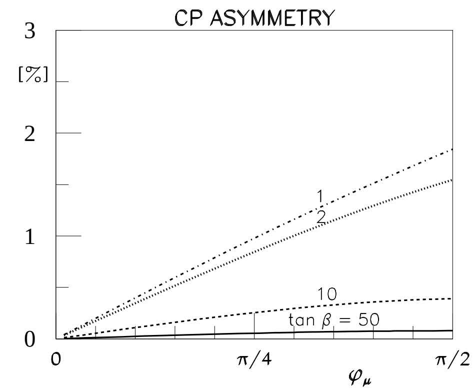

FIG. 5: Same parameters as in Fig. 4, except $\sqrt{s} = 800$ GeV.

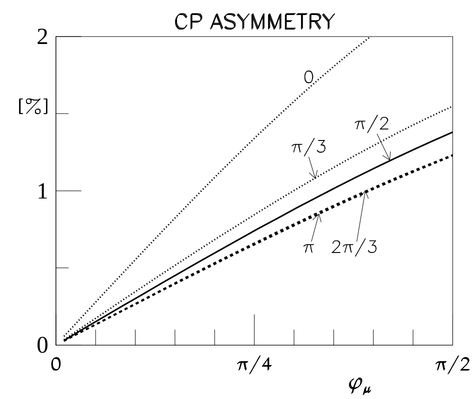

FIG. 6: The ratio (2.5) for various values of the polar angle θ. The other parameters are fixed as $\tan\beta = 5$, $\sqrt{s} = 800$ GeV.

it would vanish in the limit of equal chargino masses. This is not the case. First of all, because of the finite W mass, there is a minimum splitting among the two chargino masses. The splitting would only vanish in the limit of μM_2 being real and negative, in which case there is no CP violation. Secondly, these coupling constants do not correlate very well with the mass difference, $\Delta m = m_{\chi_2} - m_{\chi_1}$. This is illustrated in Fig. 7, where we show the quantity (4.2) vs. Δm, for the cases of fixed M_2 and fixed $|\mu|$, scanning over the other, and two values of $\tan\beta$.

V. CONCLUDING REMARKS

Since triangle diagrams have not been calculated, the results given in Figs. 4–6 are not to be seen as quantitative results, they are of a purely illustrative character.

[10] For many algebraic manipulations REDUCE and MATLAB packages were used.

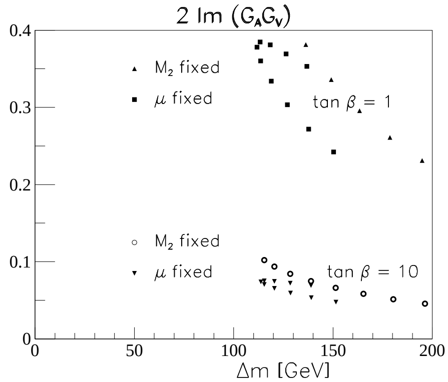

FIG. 7: The couplings of Eq. (4.2) vs. Δm, for two values of $\tan\beta$. Some points are obtained with fixed $|\mu| = 300$ GeV and varying M_2, others are obtained with fixed $M_2 = 200$ GeV and varying $|\mu|$. In all cases $\phi_\mu = \pi/2$.

However, since the kinematic structure of the triangle diagrams is different from that of the box diagrams, when included, these can not cancel the contributions of the box diagrams. Thus, we conclude that the CP-violating asymmetry in the unpolarized cross section is non-zero. Furthermore, we believe the effect, which depends on the phases of both μ and M_1, to be of the order of a percent.

To obtain the complete one-loop result for the observable (2.3), one will also need to compute the (triangle) loop corrections for each of the tree-level vertices appearing in diagrams Fig. 1 **b**, **c** and **d** (the Higgs coupling is negligible) assuming, of course, that higgsino and U(1) gaugino mass parameters are complex. Such a calculation would involve 40–60 diagrams (depending on how one counts [24]), and the heavy-sneutrino limit does not lead to any obvious simplification.[11] In contrast to the box diagrams, individual triangle diagrams are divergent. However, as was argued above, since there is no contribution to the asymmetry (2.3) at the tree level, they have to combine to a finite quantity. What is interesting to note is that this calculation may require the one-loop $\gamma\chi\chi$ vertex, absent at tree level. Indeed, the $U(1)$ gauge invariance together with renormalizability protects the photon from coupling with two fermions of different mass (see, e.g., [29]). However, the gauge invariance *alone* cannot guarantee it: for example, the (non-renormalizable) vertex

$$\left(\bar{\psi}_{\chi_1}\sigma_{\mu\nu}\psi_{\chi_2} + \bar{\psi}_{\chi_2}\sigma_{\mu\nu}\psi_{\chi_1}\right) F^{\mu\nu}, \tag{5.1}$$

where $\sigma_{\mu\nu} = \frac{i}{4}\left[\gamma_\mu, \gamma_\nu\right]$ and $F^{\mu\nu}$ is the field-strength tensor, provides such a coupling, being explicitly gauge invariant.[12] Hence, the (triangle) loop corrections to the $\gamma\chi_i\chi_j$ vertex can in principle give a (UV-finite) contribution to $d\sigma_0$ and, possibly, to $d\sigma_0^{\rm odd}$. This has to be checked. Unfortunately, the authors who recently reported complete one-loop calculations with real couplings (CP-even case) do not comment on this [23, 24, 25, 26].

If the heavier chargino is considerably heavier than the lighter one, it might be easier to observe CP-violation in the production of equal-mass charginos, using polarized beams [39], however the one-loop corrections to polarized amplitudes require a separate study.

[11] We have recently been informed that such a calculation is in progress by J. Kalinovski and K. Rolbiecki [40, 41].

[12] Couplings of this kind have been discussed in the context of neutrino propagation, see, e.g., [42].

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